Announcements

1) HW #1 up - 2 parts
Webwork + 1 problem to be handed in, under "Assignments" on CTools.

2) Practice problems up - under "Assignments" on CTools

3) Webwork "email instructor" button - never use this!

4) Office hours poll - will be closed Sunday night ~ 10 pm

Action of Matrices on Vectors

Let $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$ be an m-vector.

Let A be the matrix determined by m n-vectors

$$a_1 = \begin{bmatrix} a_{1,1} \\ a_{2,1} \\ \vdots \\ a_{n,1} \end{bmatrix}, a_2 = \begin{bmatrix} a_{1,2} \\ a_{2,2} \\ \vdots \\ a_{n,2} \end{bmatrix}, \ldots, a_m = \begin{bmatrix} a_{1,m} \\ a_{2,m} \\ \vdots \\ a_{n,m} \end{bmatrix}$$

($a_{i,j} = i^{th}$ row, j^{th} column entry).

A is an $n \times m$ matrix,

A acts on v to get an n-vector

$$w = Av = \begin{bmatrix} \sum_{i=1}^{m} a_{1,i} v_i \\ \sum_{i=1}^{m} a_{2,i} v_i \\ \vdots \\ \sum_{i=1}^{m} a_{n,i} v_i \end{bmatrix} \begin{array}{l} \to \text{number} \\ \to \text{number} \\ \\ \to \text{number} \end{array}$$

Example 1: $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 5 \\ -8 & 3 \end{bmatrix}$$

$$Av = \begin{bmatrix} 2 \cdot 1 + 5 \cdot 2 \\ -8 \cdot 1 + 3 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ -2 \end{bmatrix}$$

$$v = \begin{bmatrix} -1 \\ -5 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 10 & -6 & 4 \end{bmatrix} \quad (2 \times 3)$$

$$Av = \begin{bmatrix} -1 \cdot 4 - 5 \cdot 3 + 2 \cdot 1 \\ 10 \cdot (-1) - 6 \cdot (-5) + 4 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} -17 \\ 28 \end{bmatrix}$$

How to use a matrix to solve systems of linear equations

Given $Ax = b$, write the "augmented" matrix

$$\begin{bmatrix} A & b \end{bmatrix}$$

and row-reduce (add scalar multiples of the rows to each other) until the matrix is in row-reduced echelon form (rref)

RREF (p.13)

leading entry = first nonzero entry in a row from the left.

1) All nonzero rows are above any rows of all zeros

2) The leading entry in any nonzero row is one (leading one)

3) Each leading one is the only nonzero entry in its column.

4) Each leading one is in a column to the right of the leading one in the row above it.

Example 2: our problem

from last time:

$$x + y = 15$$
$$2x - y = 3$$

Turn into a matrix equation

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} 15 \\ 3 \end{bmatrix}}_{b}$$

$$A \quad v = b$$

Check this is right:

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot x + 1 \cdot y \\ 2 \cdot x - 1 \cdot y \end{bmatrix} = \begin{bmatrix} x + y \\ 2x - y \end{bmatrix}$$

If we set equal to $\begin{bmatrix} 15 \\ 3 \end{bmatrix}$,

then $x + y = 15$ ✓ and

$2x - y = 3$ ✓

Write the augmented matrix

$$\begin{bmatrix} 1 & 1 & 15 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\underbrace{\begin{matrix} 2 & -1 \end{matrix}}_{A} \quad \underbrace{3}_{b}$$

Put this in RREF.

<u>Step 1</u>: Row 2 added to row 1

$$\begin{bmatrix} 3 & 0 & 18 \\ 2 & -1 & 3 \end{bmatrix}$$

Step 2: Divide row 1 by 3

$$\begin{bmatrix} 1 & 0 & 6 \\ 2 & -1 & 3 \end{bmatrix}$$

Step 3: Multiply row 1 by -2, add to row 2

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & -1 & -9 \end{bmatrix}$$

Step 4: Multiply row 2 by -1.

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 9 \end{bmatrix},$$ this

reads that

$x = 6, \quad y = 9$ ✓

Example 3:

$$2x - y + z = 10$$
$$x + 7y - 8z = 2$$

Write as

$$\underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 1 & 7 & -8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} 10 \\ 2 \end{bmatrix}}_{b}$$

$$A \quad v \quad = \quad b$$

Augmented matrix

$$\begin{bmatrix} 2 & -1 & 1 & 10 \\ 1 & 7 & -8 & 2 \end{bmatrix}$$

Step 1: $-1 \cdot$ row 2 added to row 1

$$\begin{bmatrix} 1 & -8 & 9 & 8 \\ 1 & 7 & -8 & 2 \end{bmatrix}$$

Step 2: $-1 \cdot$ row 1 added to row 2

$$\begin{bmatrix} 1 & -8 & 9 & 8 \\ 0 & 15 & -17 & -6 \end{bmatrix}$$

Step 3! Divide row 2 by 15

$$\begin{bmatrix} 1 & -8 & 9 & 8 \\ 0 & 1 & -17/15 & -2/5 \end{bmatrix}$$

Step 4: 8 · row 2 added to row 1

$$\begin{bmatrix} 1 & 0 & -1/15 & 24/5 \\ 0 & 1 & -17/15 & -2/5 \end{bmatrix}$$

What this says:

$$x - \frac{z}{15} = 24/5$$

$$y - \frac{17z}{15} = -2/5$$

so $x = 24/5 + z/15$

$y = -2/5 + \frac{17z}{15}$

Since z is unrestricted,

there are infinitely many solutions;

One such solution is

when $z=0$, then

$$x = 24/5, \quad y = -2/5.$$

We can write solutions in vector form as

$$\begin{bmatrix} 24/5 \\ -2/5 \end{bmatrix} + z \begin{bmatrix} 1/15 \\ 17/15 \end{bmatrix}$$

Example 4:

$$4x_1 - x_2 + 2x_3 - 8x_4 = 10$$

$$3x_2 + 24x_4 = -15$$

$$16x_1 + 8x_3 = 28$$

$$7x_1 + x_2 - x_3 = 22$$

Augmented matrix

$$\begin{bmatrix} 4 & -1 & 2 & -8 & 10 \\ 0 & 3 & 0 & 24 & -15 \\ 16 & 0 & 8 & 0 & 28 \\ 7 & 1 & -1 & 0 & 22 \end{bmatrix}$$

RREF gets

$$\begin{bmatrix} 1 & 0 & 0 & -8/9 & 0 \\ 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16/9 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

last row says

$0 = 1$, no solution